

13+ SCHOLARSHIP EXAMINATION 2017

MATHEMATICS I

TIME ALLOWED: 90 minutes

You may use a calculator.

The marks available for each question are printed in square brackets.

This paper is divided into two sections:

Section A is worth 40 marks and contains eight questions. You should attempt all questions in Section A.

Section B is worth 60 marks and contains five questions each worth 12 marks. You may attempt all questions. Start with the ones that interest you most; answer as many questions as you can. You may find some easier than others.

Start each question from Section B on a fresh sheet of paper and put the pages in question order at the end of the exam.

Credit will be given for the clarity of your work and your explanations.

## Section A [40 marks]

1. Simplify
	1.  [1]
	2.  [2]
	3.  [2]
2. Expand and simplify
	1.  [2]
	2.  [2]
	3.  [1]
3. Solve, leaving your answers as fractions where appropriate
	1.  [1]
	2.  [2]
	3.  [2]
4. Factorise fully
	1.  [1]
	2.  [2]
	3.  [2]
5. If  and  find
	1. the product of the numbers *a*, *b*, and *c*; [1]
	2. the difference between *a* and *c*; [1]
	3. the mean of the numbers; [1]
	4.  [1]
	5.  [1]
6. Simplify the ratio 45 : 80. [1]
	1. Divide 90 in the ratio 2 : 3. [2]
	2. I share some sweets in the ratio 2 : 5. One person gets 18 sweets more than the other.
	Find the total number of sweets. [2]
7. Find 15% of 80. [1]
	1. Increase 500 by 28%. [2]
	2. Reduce 700 by 32%. [2]
8. Write down
	1. a square number between 150 and 200; [1]
	2. a fraction equivalent to  with a denominator between 50 and 60. [2]
	3. a prime number greater than 30 that ends in a 3; [1]
	4. a two-digit factor of 91 (that is not equal to 91). [1]

## Section B – Problems [60 marks]

1. A new currency system is proposed. There will just be one unit of currency, the Welling, and the only coins that will be produced will be the 18 Welling coin and the 59 Welling coin.
*(To “pay exactly” means to give the right amount without needing any change.)*
	1. How would you pay exactly for an object costing 72 Wellings? [1]
	2. How would you pay exactly for an object costing 172 Wellings? [2]
	3. I want to buy an item costing 23 Wellings. I pay with a single 59 Welling coin.
	How much change am I owed, and how could I be given the change? [2]
	4. I want to buy an item costing 28 Wellings.
	Show that if I use only one 59 Welling coin then I can’t be given the correct change, but that if I use two 59 Welling coins, I can be given the correct change. [2]
	5. I want to pay for an item costing exactly 1 Welling. How should I pay for it so I can get the right change? [5]
2. I often do rough work on squared paper. Sometimes when I find it hard to come up with good ideas I doodle on the paper.
One morning I start colouring in the following pattern of squares:

Each doodle consists of some shaded squares and some short black edges.
The first doodle is made up of four shaded squares and twelve edges.

* 1. Write down the number of shaded squares and the number of edges in the second doodle. [2]
	2. Work out how many shaded squares and edges there would be in the 10th doodle. [3]
	3. Find a formula for the number of shaded squares and the number of edges in the *n*th doodle. [3]
	4. Determine which doodle uses 228 edges. [1]
	5. After many hours of drawing, I produce a doodle where the number of edges is 1000 bigger than the number of shaded squares.
	How many shaded squares make up the doodle? [3]
1. (a) I take three books from my shelf, *Harry Potter and the Chamber of Secrets*, *Harry Potter and
 the Goblet of Fire*, and *Harry Potter and the Deathly Hallows*.

In total, the three books have 1494 pages.
*Goblet of Fire* is 29 pages longer than *Deathly Hallows*.
It is also only 117 pages short of being three times the length of C*hamber of Secrets*.

Find the length of each book. [7]

(b) The staff of Gringotts, the famous wizarding bank, decide to relocate from London to Switzerland.
900 Swiss Francs is worth exactly 117 Galleons and 11 Sickles.
450 Swiss Francs is worth exactly 58 Galleons and 14 Sickles.

Find the value of a Galleon and a Sickle in Swiss Francs. [5]

1. The Lazy Ruler Company sells rulers at a variety of different lengths. Unlike most rulers, they don’t have markings at every centimetre.
Here is their 3 cm ruler:

It only has a mark at 1 cm. They say this is enough, because you can measure 1 cm to the left of the line, or 2 cm to the right of the line, or 3 cm using the whole ruler.

Here is their 4 cm ruler, which has marks at 1 cm and 2 cm but not at 3 cm:

A

B

C

D

* 1. Use the letters to list a way in which you could measure 1 cm, 2 cm, 3 cm and 4 cm using this ruler. [2]

The company describes their rulers as “complete” if it is possible to measure all lengths up to the maximum.

* 1. This 5 cm ruler has two marks: one at 1 cm and one at 4 cm. Explain why it is not complete.
	 [1]
	2. Suggest a suitable way of making a complete 5 cm ruler using only two marks. [1]
	Show how to make all the lengths needed using your ruler. [1]
	3. Show that it is possible to make a complete 6 cm ruler with only two marks. [3]
	4. Show that it is not possible to make a complete 8 cm ruler with only two marks, but that it is possible with three marks. [4]

 *Question 13 appears on the next page*

1. Two identical rectangles, ABCD and PQRS, are arranged so one sits partially overlapping the other, as illustrated. D lies on the line PS. The diagram is not to scale.

SR measures 187 cm and XC measures 84 cm.
PD measures 156 cm.
The perimeter of the overlapped shape, ABXQRSD, is 1388 cm.

A

B

C

D

P

Q

R

S

X

* 1. Show that DX measures 205 cm. [2]
	2. Hence find PX and XQ. [3]
	3. Let the length of QR be *x*.
	Find *x*. [3]
	4. Find the total area of the overlapped shape, ABXQRSD. [4]

**END OF QUESTIONS**