

13+ SCHOLARSHIP EXAMINATION 2017

MATHEMATICS II

TIME ALLOWED: 90 minutes

You may not use a calculator in this examination.

The marks available for each question are printed in square brackets.

This exam has seven questions. Each is worth 12 marks.

Start each question on a fresh sheet of paper and put the pages in question order at the end of the exam.

Credit will be given for the clarity of your work and your explanations.

1. (a) Explain why, if *n* is an integer,  will be an odd number. [1]

(b) Show algebraically that the square of an odd number will always be an odd number. [3]

(c) Show that the product of two consecutive odd numbers will always be one less than a multiple of four. [4]

(d) Show that the sum of the squares of two consecutive odd numbers will always be even but will never be a multiple of four. [4]

1. In this question, you should briefly explain your reasoning in each part.   
   Only three marks are available in total for the correct answers: the rest are awarded for showing and explaining your method.
   1. I write down all the numbers from 1 to 100 in the usual way.  
      How many times do I write the number 7? [4]
   2. I write down all the numbers from 1 to 1000 in the usual way.  
      How many times do I write the number 7? [5]
   3. I write down all the numbers from 1 to 2017 in the usual way.   
      How many times do I write the number 7? [3]
2. Sometimes it is easier to use algebra than numbers, even if a problem seems to be completely numerical. For example, if you are asked to simplify  one way to do it would be to let  and write the following:



Which is easier than trying to work out  and  and subtracting.

* 1. Show that if ** you canrewrite  as  [3]
  2. Hence find the value of  [2]
  3. Choose a suitable value of *n* and hence find  [4]
  4. Use a similar method to find  [3]

1. The volume of a sphere of radius *r* is and its surface area is    
   A plastic bollard is made of a solid cylinder and a solid hemisphere as illustrated.  
      
   The diameter of its base is 20 cm and its total surface area, including the base, is cm2.
   1. Find the height of the cylindrical section of the bollard. [4]
   2. Hence find the volume of the bollard. Leave your answer as a multiple of  [3]
   3. An engineer suggests that the bollard would be cheaper, lighter and safer if it were hollow instead of solid. The design is modified so that the plastic is 1 cm thick the whole way round.  
      Find the volume of plastic required to make the hollow bollard.   
      Leave your answer as a multiple of  [5]
2. *a* and *b* are both integers (whole numbers).
   1. Write down all the possible solutions to the equation  [4]
   2. Show clearly that the equation   
         
      may be written in the form  
        [2]
   3. Given that *c* and *d* are both positive integers, use parts (a) and (b) to solve the equation   
        [6]

*Questions 6 and 7 appear on the next page*

1. Pascal’s triangle is an arrangement of numbers in a triangular grid. Each entry in the grid is the sum of the numbers directly above it. For example, each “10” in row 6 comes as a result of the 4 and the 6 directly above it

1

1

1

2

1

3

1

4

1

5

1

Row 1

1

1

1

3

4

6

10

10

1

5

6

…

1

Row 2

Row 3

Row 4

Row 5

Row 6

Row 7

* 1. Write down the numbers that would complete row 7. [2]
  2. Write down row 8. [2]
  3. Predict the value of the second cell in row 50. [1]
  4. Find the total of the cells in row 2, the total of the cells in row 3 and the total of the cells in row 4. [1]
  5. Predict the total of the cells in row 11. [2]
  6. Investigate the values in the third cell of each row.  
     Predict the value found by adding the third cell in row 101 to the third cell in row 102.  
     Explain how you have reached this prediction. [4]

1. Table tennis balls have a diameter of 40 mm and a mass of 2.7 g.  
   Wembley Stadium seats around 90,000 people. The average mass of a person is 62 kg.  
   The interior of the stadium (‘the bowl’) has a volume of roughly 1.1 million cubic metres.  
   Use suitable approximations to decide which has a greater mass:
   1. the stadium when it has its full audience of people, or
   2. the stadium when the bowl has been filled to the brim with table tennis balls? [1]

Justify your answer clearly with calculations. [9]

Roughly how many times greater is the larger mass than the smaller mass?  
Give your answer to a sensible degree of accuracy. [2]

**END OF QUESTIONS**